

Extending characterizations of truthful mechanisms from subdomains to domains

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Mechanism Design

Social Choice: A choice for the whole society

We need to construct a function that takes as input the **preferences of many different individuals** and “amalgamates” (/aggregates) them in a **single** preference or choice.

Mechanism Design

Design a game whose **outcome** is an **equilibrium** for the players.

Amalgamates here means: no player can gain by deviating

Truthful mechanisms

Selfish Players want to maximize their Utility

$$u_i(a) := v_i(a_i) - p_i(v_i, v_{-i})$$

Definition (Truthful mechanisms "*A player does not gain by lying.*")

A mechanism is truthful if revealing the true values is a dominant strategy of each player. For all v_i, v'_i, v_{-i} , if $f(v_i, v_{-i}) = a$ and $f(v'_i, v_{-i}) = a'$ then $v_i(a_i) - p_i(v_i, v_{-i}) \geq v_i(a'_i) - p_i(v'_i, v_{-i})$

A success story: A non-manipulable mechanism!

The VCG [Vickrey, Clarke, Groves] auction



A single item for sale:

valuation of player 1: 10 The player with the highest bid **wins**.

valuation of player 2: 3

valuation of player 3: 8 ←and **pays** the second-highest bid.

- No player can gain by lying. (non-manipulable, truthful)

The objective is to maximize the social welfare. Selfish players are utility maximizers. Here the payments are such, that **the utility of all players is the social welfare!**

Affine maximizers

a direct generalization of the VCG which is still non-manipulable

The VCG Mechanism

Select an allocation that maximizes the sum of the valuations $\sum_i v_i(a_i)$.

Affine maximizers

A mechanism is an affine maximizer if there are constants $\lambda_i > 0$ (one for each player i) and γ_a (one for each of the n^m allocations) such that the mechanism selects the allocation a which maximizes $\sum_i \lambda_i \cdot v_i(a_i) + \gamma_a$.

$$\begin{array}{lcl} \text{player 1} & \lambda_1 \cdot & \rightarrow v_1(a_1) + \\ \text{player 2} & \lambda_2 \cdot & \rightarrow v_2(a_2) + \gamma_a \end{array}$$

Any rival to the VCG mechanism?

Two characterization theorems in one

Truthful=non-manipulable [the Revelation Principle]

Gibbard-Satterwaite theorem for voting rules (1973)

For 3 or more outcomes, the only truthful mechanism is **dictatorship**.

Robert's theorem (1979)

For 3 or more outcomes, allowing payments, if we suppose that the domain of valuations is **unrestricted** the only truthful mechanisms are the **affine maximizers**.

You can use Robert's as a black box to get Gibbard-Satterwaite:

The only **affine maximizers** without payments are **dictatorships**...

Open questions,

which we will answer partially for the 2-player case.

Unrestricted valuations are unrealistic.

- Characterize more realistic domains like combinatorial auctions!
- How much do we need to **restrict the domain in order to admit mechanisms different than affine maximizers?**
- Use a **unified proof** for characterizing different domains!
- Use the characterization theorem for one domain as a black box to obtain characterizations of other domains!

Combinatorial auction

There are n buyers (/players) and m different items for sale. The valuation of a player does not depend on the allocation of other players.

Protocol

- The players declare their valuations
- The mechanism determines an **allocation** and **payments**
 - it allocates **all** items
 - the payments are based: on the declared valuations & on the allocation

Objective of a selfish player: $\text{maximize}\{\text{utility}\}$

$\text{utility} = \text{valuation} - \text{payment}$ (we assume here quasilinear utilities)

Objective of the mechanism designer

We want to find out **all possible objectives** that are truthfully implementable.

Each one of these domains is a subdomain of the previous
[1/2]

- Unrestricted valuations (Robert's domain) a, b, c, d outcomes.

$$\begin{array}{l} \text{player 1} \\ \text{player 2} \end{array} \begin{pmatrix} v_1(a) & v_1(b) & v_1(c) & v_1(d) \\ v_2(a) & v_2(b) & v_2(c) & v_2(d) \end{pmatrix}$$

- Combinatorial public projects: the valuations are *submodular* and $v_i(\emptyset) = 0$. (The valuations are restricted but the outcome is the same for all players just like in the previous domain.)

$$\begin{pmatrix} v_1(\emptyset) = 0 & v_1(\{1\}) & v_1(\{2\}) & v_1(\{1, 2\}) \\ v_2(\emptyset) = 0 & v_2(\{1\}) & v_2(\{2\}) & v_2(\{1, 2\}) \end{pmatrix}$$

Each one of the domains is a superdomain of the previous [2/2]

The possible outcomes are

$$a = \{\emptyset, \{1, 2\}\}, b = \{\{1\}, \{2\}\}, c = \{\{2\}, \{1\}\}, d = \{\{1, 2\}, \emptyset\}$$

- Combinatorial auctions: the valuations are *submodular* or *subadditive* or *superadditive* or *additive* and each item is allocated to exactly one player.

$$\begin{pmatrix} v_1(\emptyset) = 0 & v_1(\{1\}) & v_1(\{2\}) & v_1(\{1, 2\}) \\ v_2(\{1, 2\}) & v_2(\{2\}) & v_2(\{1\}) & v_2(\emptyset) = 0 \end{pmatrix}$$

- Additive/Scheduling Domain

$$\begin{pmatrix} v_1(\emptyset) = 0 & v_1(\{1\}) & v_1(\{2\}) & v_1(\{1\}) + v_1(\{2\}) \\ v_2(\{2\}) + v_2(\{1\}) & v_2(\{1\}) & v_2(\{2\}) & v_2(\emptyset) = 0 \end{pmatrix}$$

Unrestricted Valuations $\subseteq$ Combinatorial public projects $\subseteq$ Combinatorial auctions $\subseteq$ Additive combinatorial auction

Most common restrictions on the valuations

If A, B are two sets of items:

- Free disposal: $A \subseteq B$ we have that $v_i(A) \leq v_i(B)$ ✓ [LMN. FOCS '03]
- Subadditivity: $v_i(A) + v_i(B) \geq v_i(A \cup B)$ ✓ [DS, EC '08]
- Supperadditivity: $v_i(A) + v_i(B) \leq v_i(A \cup B)$ ♠
- Submodularity: $v_i(A) + v_i(B) \geq v_i(A \cup B) + v_i(A \cap B)$ ♠
- Additivity: $v_i(A) + v_i(B) = v_i(A \cup B)$ ✓ [CKV, ESA '08]

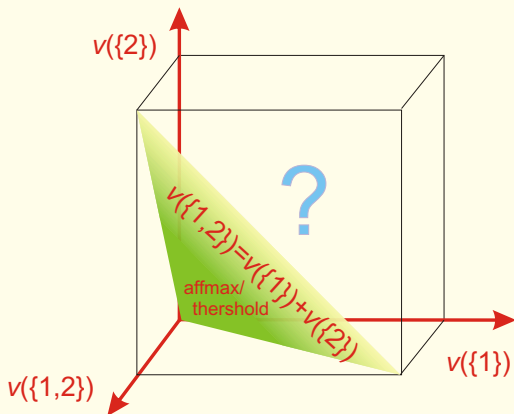
✓: a characterization was known for the case of 2 players

♠: we give a characterization for the case of 2 players here

We give a unique characterization proof for ✓s and ♠s as well as all combinatorial auctions that are superdomains of a slight perturbation of additive cominatorial auctions.

The domain and the subdomain

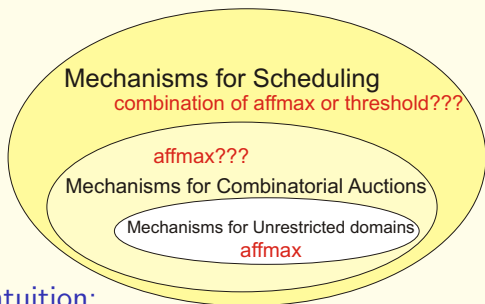
We know the characterization for the shaded subdomain.



Does the characterization hold for the whole domain? (If the mechanism wasn't truthful there would exist many possible ways to extend the mechanism to the big domain.)

Comparing Characterizations for different domains

Is scheduling harder than combinatorial auctions or is it the other way around?



Intuition:

The richer the domain, the bigger the input space, the more restrictive truthfulness becomes, the fewer are the possible algorithms, the less difficult a characterization.

Can we give a rigorous proof of this intuition?

some broken proofs

- We have to give a different proof for each domain we can just borrow some ideas from one domain to another.
- **NO!**
- It is obvious: The same characterization applies: since we used truthfulness for a subset of the possible valuations (input) and the only truthful mechanisms were affine maximizers then the only possible mechanisms for the bigger domain are affine maximizers too.
- Well, this gives that the restriction of the mechanism to the smaller domain is an affine maximizer. But we don't know if the mechanism an affine maximizer for the whole domain.

Characterizations

Theorem (Roberts, '79)

*For the **unrestricted domain** with at least 3 outcomes, the only truthful mechanisms are affine maximizers.*

Theorem (Lavi, Mu'alem and Nisan, FOCS '03)

*For n -player combinatorial auctions that satisfy **free disposal** and **very large input S-MON** (which can be removed for the 2-player case) the only decisive truthful mechanisms are affine maximizers.*

Theorem (Dobzinski, Sundararajan EC '08)

*For **2-player subadditive** combinatorial auctions with the only truthful mechanisms are affine maximizers.*

Derivation of the characterization of a domain from the characterization of one of its subdomains

Theorem

*Let V be a subdomain of the 2-player combinatorial auctions that allocate all items (n -player S -MON combinatorial auctions). If the only possible mechanisms for V are *affine maximizers*, then the same holds *for every superdomain* of V . (we do not assume decisiveness!)*

- We would like to apply this theorem and **use additive combinatorial auctions as the subdomain**. (All other domains we are interested in are superdomains of this domain.)
- Unluckily affine maximizers are not the only mechanisms for this domain. (also **threshold** mechanisms are possible)

Our black box

Theorem (Christodoulou-Koutsoupias-Vidali ESA '10)

*For 2-player **additive** combinatorial auctions (/2-player scheduling), the decisive truthful mechanisms that allocate all items partition the items in groups allocated by **threshold mechanisms** or **affine maximizers**.*

Scheduling unrelated machines

[Algorithmic Mechanism Design, Nisan and Ronen FOCS'99]

The matrix of processing times

We want to process m tasks using n machines(/selfish players).

We have the following matrix of processing times:

	task 1	...	task j	...	task m
player 1	t_{11}	...	to process task j needs time t_{ij}	...	t_{1m}
$\vdots$					
player i					
$\vdots$					
player n	t_{n1}			$\ddots$	t_{nm}

The players **get payed in order to process the tasks.**

Scheduling unrelated machines

[Algorithmic Mechanism Design, Nisan and Ronen FOCS'99]

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We have the following matrix of processing times:

	task 1	...	task j	...	task m
player 1	t_{11}	...			t_{1m}
$\vdots$					
player i					
$\vdots$					
player n	t_{n1}				t_{nm}

player i needs time

to process t_{ij}

*Only player i knows
the values of his line.
He can report a false value!*

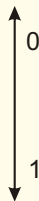
The players get paid in order to process the tasks.

Auctions or Scheduling?

The world upside down



Auctions



Scheduling

- Auction: sell the objects to bidder who values them **high**
- **Scheduling**: allocate the task to machines with **small** processing times
Change **max** → **min**

The VCG [Vickrey, Clarke, Groves] mechanism

Combinatorial auction

Possible Outcomes:	$\left\{ \text{only } \img alt="Mona Lisa" data-bbox="421 168 491 248" \right\}$	$\left\{ \text{only } \img alt="Mona Lisa" data-bbox="628 168 683 248" \right\}$	$\left\{ \text{both } \img alt="Mona Lisa" data-bbox="825 168 890 248" \img alt="Mona Lisa" data-bbox="898 168 953 248" \right\}$
Valuation of player 1:	10	6	10
valuation of player 2:	3	5	8
valuation of player 3:	2	9	20

- Goal achieved: **maximize the sum of the valuations**

Scheduling (Essentially a combinatorial auction with additive vauations!)

Possible Outcomes:	$\left\{ \text{only } \img alt="Mona Lisa" data-bbox="421 554 491 634" \right\}$	$\left\{ \text{only } \img alt="Mona Lisa" data-bbox="628 554 683 634" \right\}$	$\left\{ \text{both } \img alt="Mona Lisa" data-bbox="825 554 890 634" \img alt="Mona Lisa" data-bbox="898 554 953 634" \right\}$
Valuation of player 1:	10	6	10+6
valuation of player 2:	3	5	3+5
valuation of player 3:	2	9	2+9

- Goal achieved: **minimize the sum of processing times**
- We don't need the last column because it is always the sum.

The VCG [Vickrey, Clarke, Groves] mechanism

Combinatorial auction

Possible Outcomes:	{only 	{only 	{both  
Valuation of player 1:	10	6	10
valuation of player 2:	3	5	8
valuation of player 3:	2	9	20

- Goal achieved: **maximize the sum of the valuations**

Scheduling (Essentially a combinatorial auction with additive valuations!)

Possible Outcomes:	{only 	{only 	{both  
Valuation of painter 1:	10	6	10+6
valuation of painter 2:	3	5	3+5
valuation of painter 3:	2	9	2+9

- Goal achieved: **minimize the sum of processing times**
- We don't need the last column because it is always the sum.

Task-Independent and Threshold Mechanisms

Both have no sloped lines on the projections.

Definition (Task(/Item)-Independent)

Each task is allocated independently of the rest.

Definition (Threshold Mechanisms)

"Each item is allocated independently of the other values of player i ."

There are thresholds $h_{ij}(v_{-i})$ such that the player i gets task j iff the value $v_{ij} = v_i(\{j\})$ is less than h_{ij} .

Example

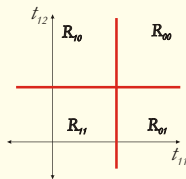
Task(/Item)-independent:

$$\begin{pmatrix} v_{11} & \mathbf{v_{12}} & v_{13} & v_{14} \\ v_{21} & \mathbf{v_{22}} & t_{23} & v_{24} \\ v_{31} & \mathbf{v_{32}} & t_{33} & v_{34} \end{pmatrix}$$

Threshold:

$$\begin{pmatrix} v_{11} & \mathbf{v_{12}} & v_{13} & v_{14} \\ \mathbf{v_{21}} & \mathbf{v_{22}} & \mathbf{v_{23}} & \mathbf{v_{24}} \\ \mathbf{v_{31}} & \mathbf{v_{32}} & \mathbf{v_{33}} & \mathbf{v_{34}} \end{pmatrix}$$

(The red items can affect the allocation a_{12} .)



Affine transformations of domains

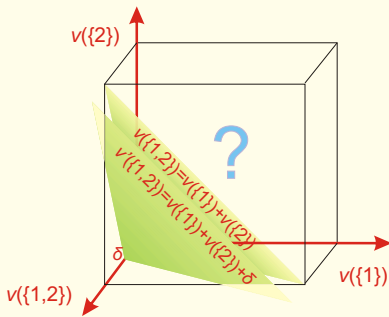
Bijection

Tell me which are the truthful mechanisms for the domain D
 I can tell you which are the truthful mechanisms for $\lambda D + \delta$!

Threshold mechanism:

For the domain D of additive valuations iff: $p_i(a_i) = \sum_{j \in a_i} p_i(\{j\})$

For the domain $\lambda D + \delta$ iff: $p_i(a_i) - \delta_{a_i} = \sum_{j \in a_i} (p_i(\{j\}) - \delta_{\{j\}})$.



Enrich slightly the possible valuations

...and the threshold mechanisms vanish!

domain S of additive valuations: $v(\{1, 2\}) = v(\{1\}) + v(\{2\})$

domain $S + \delta$ slight perturbation: $v'(\{1, 2\}) = v(\{1\}) + v(\{2\}) + \delta$

Theorem

Consider the domain where

the valuations of player 1 are from: $(S \cup (S + \delta))$ and

the valuations of player 2 are from: S ,

*then the only truthful mechanisms for **any superdomain** of it are affine maximizers.*

- Submodular, subadditive and superadditive combinatorial auctions are its superdomains. We characterized them all at once.
- Scheduling is the transition domain that admits truthful mechanisms other than affine maximizers.

Open problems

- Generalize this approach for the case of $n \geq 3$ players.
- But... can we hope for 3-player characterizations in terms of affine maximizers?