Frugal mechanisms with monitoring

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working paper
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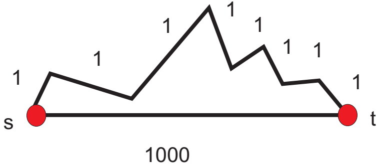
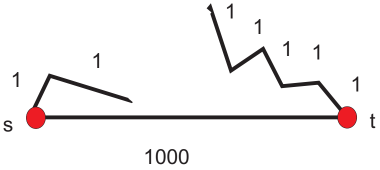
Motivation

Construct mechanisms for financing public projects where the users pay a non-extravagant amount to get serviced.

Construct mechanisms for crowdsourcing where the machines/agents get payed to process the tasks a resonable but not too high amount.

The second price auction is not always frugal





Notation

- set of outcomes $x \in \mathcal{O}$
- n selfish agents.
- $t_i(x)$ the type of agent i is the *cost* paid by i to implement x.
- t_i is *private knowledge* of agent i.
- The set of all legal cost functions t_i is the *domain* D_i of i
- $D = D_1 \times \ldots \times D_n$

A *mechanism* is a pair (f, p), where $f : D \to \mathcal{O}$ and $p : D \to \mathbb{R}^n_{\geq 0}$ that determines:
an allocation $x \in \mathcal{O}$
and payments $p = (p_1, \ldots, p_n)$.
For mechanism (f, p) let $u_i^{(f,p)}(b_i, \mathbf{b}_{-i})$ denote the *utility* of agent i for the output computed by (f, p) on input $(b_i, \mathbf{b}_{-i})$.

Truthful and Collusion-resistant mechanisms

Definition (Truthful mechanisms)

A mechanism (f, p) is *truthful* if for any i, any bids $\mathbf{b}_{-i}$ of the agents other than i, and any $b_i \in D_i$, $u_i^{(f,p)}(t_i, \mathbf{b}_{-i}) \geq u_i^{(f,p)}(b_i, \mathbf{b}_{-i})$.

No player can increase his utility by deviating.

A stronger requirement demands truthtelling to be a dominant strategy for coalitions of agents:

Definition (Collusion-resistant mechanisms)

A mechanism (f, p) is *collusion-resistant* if for any subset C of agents, any bids $\mathbf{b}_{-C}$ of the agents other than those in C, and any $\mathbf{b}_C \in \times_{i \in C} D_i$, $\sum_{i \in C} u_i^{(f,p)}(\mathbf{t}_C, \mathbf{b}_{-C}) \geq \sum_{i \in C} u_i^{(f,p)}(\mathbf{b}_C, \mathbf{b}_{-C})$, $\mathbf{t}_C$ denoting the vector $(t_i)_{i \in C}$.

No ***group*** *of players can increase their sum of utilities(/wellfare) by deviating.*

Monitoring

Definition (Mechanism with monitoring[KVW'15])

In a *mechanism with monitoring* (f, p), the bid b_i is a *lower bound* on agent i's cost of using $f_i(b_i, \mathbf{b}_{-i})$, so an agent is allowed to have a real cost higher than $b_i(f(\mathbf{b}))$ but not lower. Formally, we have $u_i^{(f,p)}(b_i, \mathbf{b}_{-i}) := p_i(\mathbf{b}) - \max\{t_i(f(\mathbf{b})), b_i(f(\mathbf{b}))\}$.

Example

- Suppose that a machine overbids and says it needs 10 minutes to process a task while its true type is 5 minutes.
  $b_i = 10, t_i = 5$ then the player experiences cost of $b_i = 10$ minutes .
- Suppose that a machine underbids and says it needs 5 minutes to process a task while its true type is 10 minutes.
  $b_i = 5, t_i = 10$ then the player experiences cost of $t_i = 10$ minutes .

The *first price auction* is collusion-resistant with monitoring.

Theorem

The "first price auction" $p_i(\mathbf{b}) = b_i(f(\mathbf{b}))$ is collusion-resistant with monitoring.

Corollary

Let Π be a utilitarian cost-minimization problem. There exists a collusion-resistant budget-feasible first-price mechanism (f, p) with monitoring and budget B if for any instance I of Π, $cost(f(I)) \leq B$.

Other mechanisms that are collusion-resistant?

Shifting the payments preserves truthfulness and collusion-resistance

$$p_i(\mathbf{b}) = b_i(f(\mathbf{b})) + h_i \qquad \text{for collusion-resistance}$$

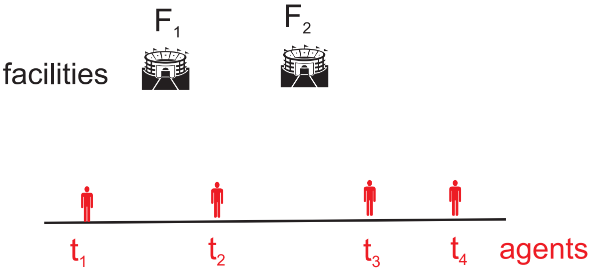
$$p_i(\mathbf{b}) = b_i(f(\mathbf{b})) + h_i(\mathbf{b}_{-i}) \qquad \text{for truthfulness}$$

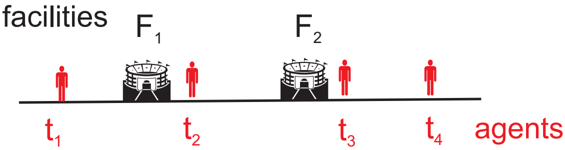
More ambitious: which are all possible collusion-resistant mechanisms?

Definition (NPT)

We say that a mechanism satisfies the *No Positive Transfer (NPT)* property if $p_i(\mathbf{b}) \geq 0$ for all i and $\mathbf{b}$.

Case study: facility location





Facility location

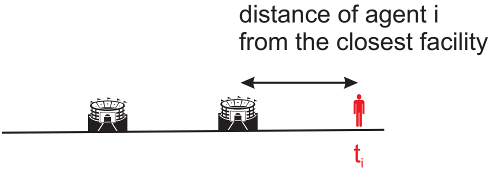
GOAL: optimize the *social cost*

COST OF EACH AGENT: Agent i pays a connection fee p_i as we saw before the first price auction ($p_i = b_i$) is collusion-resistant.

$$t_i(f(t_i, \mathbf{b}_{-i})) = |t_i - f_{t_i}(t_i, \mathbf{b}_{-i})|$$

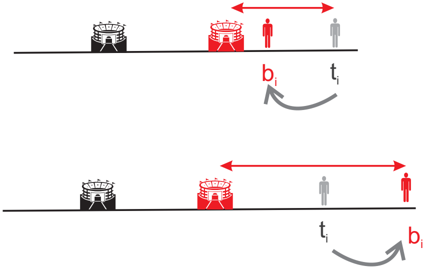
where $f_{t_i}(t_i, \mathbf{b}_{-i})$ denotes the location of the facility output by the mechanism $f(t_i, \mathbf{b}_{-i})$ closer to location t_i

In other words, $t_i(f(t_i, \mathbf{b}_{-i}))$ denotes the distance between t_i and the location of $f_{t_i}(t_i, \mathbf{b}_{-i})$.



Monitoring

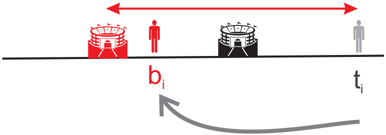
cost experienced by player with true type t_i when lying and



reporting b_i

When you lie and get connected to a different facility

When you lie and get connected to a facility wich is not the closest to you you have to stick with it cost experienced by player with true type t_i when lying and reporting b_i



Anonymity or fixed tie-breaking

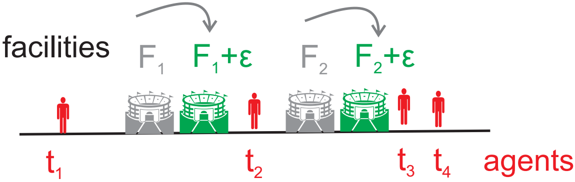
Assuming anonymity or fixed tie-breaking for facility location there is no collusion-resistent shifted first-price budget-feasible mechanism with monitoring, that uses an optimal algorithm f^* whose budget is smaller than $\max_{\mathbf{b}\in D} cost(f^*(\mathbf{b}), \mathbf{b})$.

Trade-off between approximation and frugality

Given an optimal algorithm f^*, we define f^*_ε as the algorithm that shifts the location of the facilities output by f^* by ε. Formally, $f^*_\varepsilon(\mathbf{b}) = (F_j + \varepsilon)_{j=1,\dots,K}$, $(F_1, \dots, F_k)$ denoting the output of $f^*(\mathbf{b})$. We ask whether by moving from f^* to f^*_ε the cost paid in the approximation guarantee can be compensated by a lower budget.

Theorem

There is no truthful shifted first-price budget-feasible mechanism with monitoring for facility location, that uses f^*_ε defined upon an optimal algorithm that returns lexicographically minimal (or maximal) optimal allocation whose budget is smaller than $\max_{\mathbf{b}\in D} cost(f^*_\varepsilon(\mathbf{b}), \mathbf{b})$.



Summary

We study a class of collusion-resistent mechanisms with monitoring, namely first price auction and its shifts and explore when the payments are the smallest(most frugal) under the NPT condition.
We apply our theory to facility location and also explore the tradeoff between approximability and frugality.
Open question: Characterize all possible collusion-resistent mechanisms with monitoring.