Mechanism Design for Scheduling with Uncertain Execution Time.

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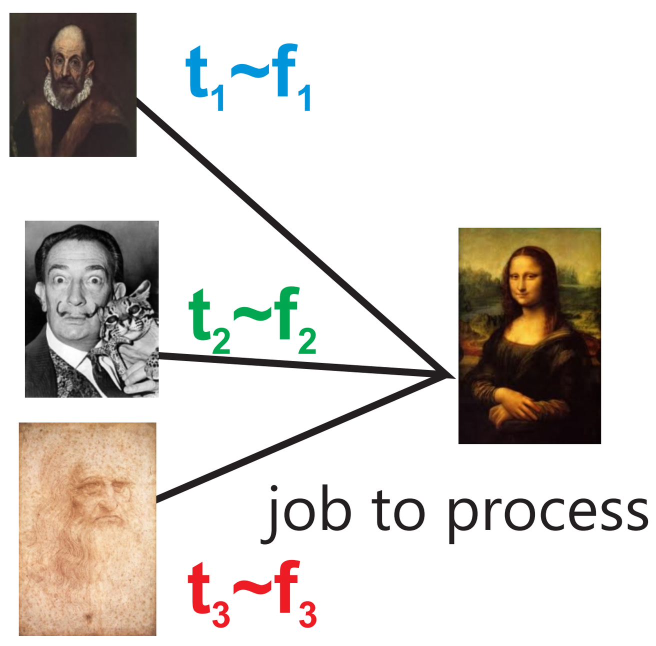
• The setting

Goal of mechanism designer:
minimize E [sum of painting times]
Every day she decides which painters will draw.

Time painter i needs to finish the job ~ distribution f_i

painter I knows the distribution f_i (this is his type) from which his painting time is drawn but not his painting time t_i or the distributions of the other players f_{-i}

Players are selfish
want to maximize their utility,
which is:
E [payment – time spent painting]



Monotone hazard rate assumption:
The probability a painter finishes the painting at time t given that he hasn't finished it until time t-1 is non-increasing.

We want a mechanism where the players have no incentive to misreport their types or miscompute.

• The efficient solution

Greedy=OPT: assign at each time step the job to the machine with maximum hazard rate, i.e. the machine most likely to finish!
To prove this we need: Monotone hazard rates assumption

OPT satisfies the Consistency Property:
"If we remove one player, to get OPT for the rest of the players we just need to remove the player from the schedule."

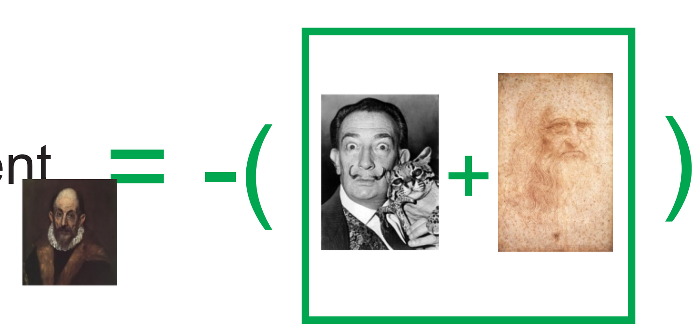
• Expected Clarke isn't truthful

The player who is most likely to finish at the first time-step has an incentive to over-report his probability of finishing at the first step.

• Groves Realized is ex-post truthful

After completing the task we have the realized running times

Groves payment



-*"sum of the realized times of the other players"*

• Solution concept: ex-post equlibrium

Valuations are interdependent: a player's utility is affected by the other players' true distributions because those will affect the probability that she gets to run.

Ex-post equilibrium If the other players are telling the truth, then the best thing for me to do is to tell the truth, for **any** private information the players might have.

Dominant $\subseteq$ Ex-post $\subseteq$ Bayes Nash

• Vickrey Variations

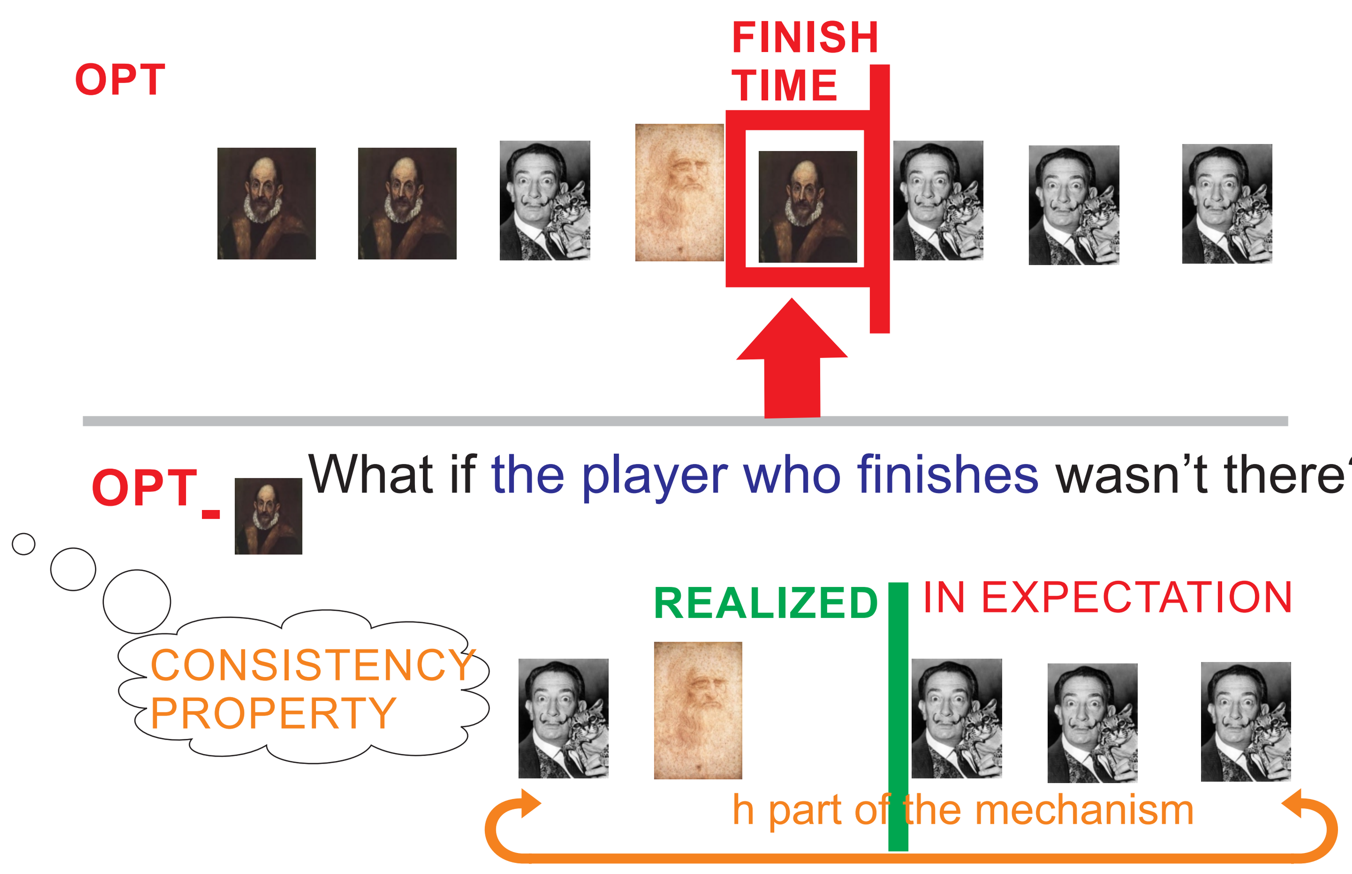
h_i(types of the other players) part

T_N :=how long it takes a group N to finish the task (random variable)
r_N :=realized value of T_N

	Groves part		$h_i()$	
$payment_i =$	$E[T_N - T_i]$	+	$E[T_{N\setminus\{i\}}]$	(CE) Clarke in Expectation
$payment_i =$	$-(r_N - r_i)$	+	0	Pure Realized Groves (PRG)
$payment_i =$	$-(r_N - r_i)$	+	$E[T_{N\setminus\{i\}}]$	(ChE) Clarke h in Expectation
$payment_i =$	$-(r_N - r_i)$	+	$(r_N - r_i) + E[T_{N\setminus\{i\}} - r_{N\setminus\{i\}} \mid T_{N\setminus\{i\}} \geq r_{N\setminus\{i\}}]$	(ChpE) Clarke h partially in Expectation

This rewritting uses the consistency property!

"How much does the player who finish contribute to the social welfare?"



• Properties of different Mechanisms

	efficient	truthful in dominant strategies	ex-post truthful	IR	no incentive to miscompute	payment 0 if fail
Clarke in Expectation (CE)	✔	✕	✕	✔	✕	✕
Pure Realized Groves (PRG)	✔	✕	✔	✕	✔	✕
Clarke h (ChE) in Expectation	✔	✕	✔	✔	✔	✕
Clarke h partially in Expectation (ChpE)	✔	✕	✔	✔	✔	✔