A characterization of n-player strongly monotone scheduling mechanisms

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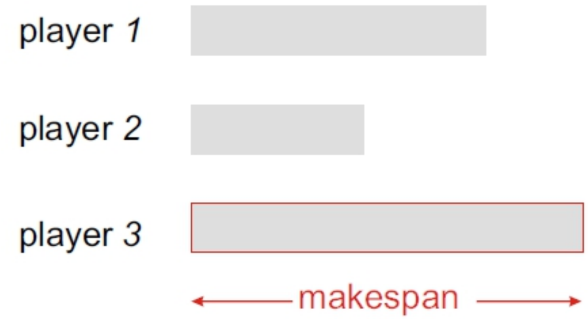
Scheduling Selfish unrelated Machines

Finish all tasks as fast as possible! You can parallelize!



Input: matrix of processing times

m tasks

n players/ machines

Player 1  needs time

Player n 

$$\begin{pmatrix} & \text{for task 1} & \text{for task 2} & \text{for task m} \\ \text{Player 1 needs time} & v_{11} & v_{12} & v_{1m} \\ & v_{21} & v_{22} & v_{2m} \\ \text{Player n} & v_{n1} & v_{n2} & v_{nm} \end{pmatrix}$$

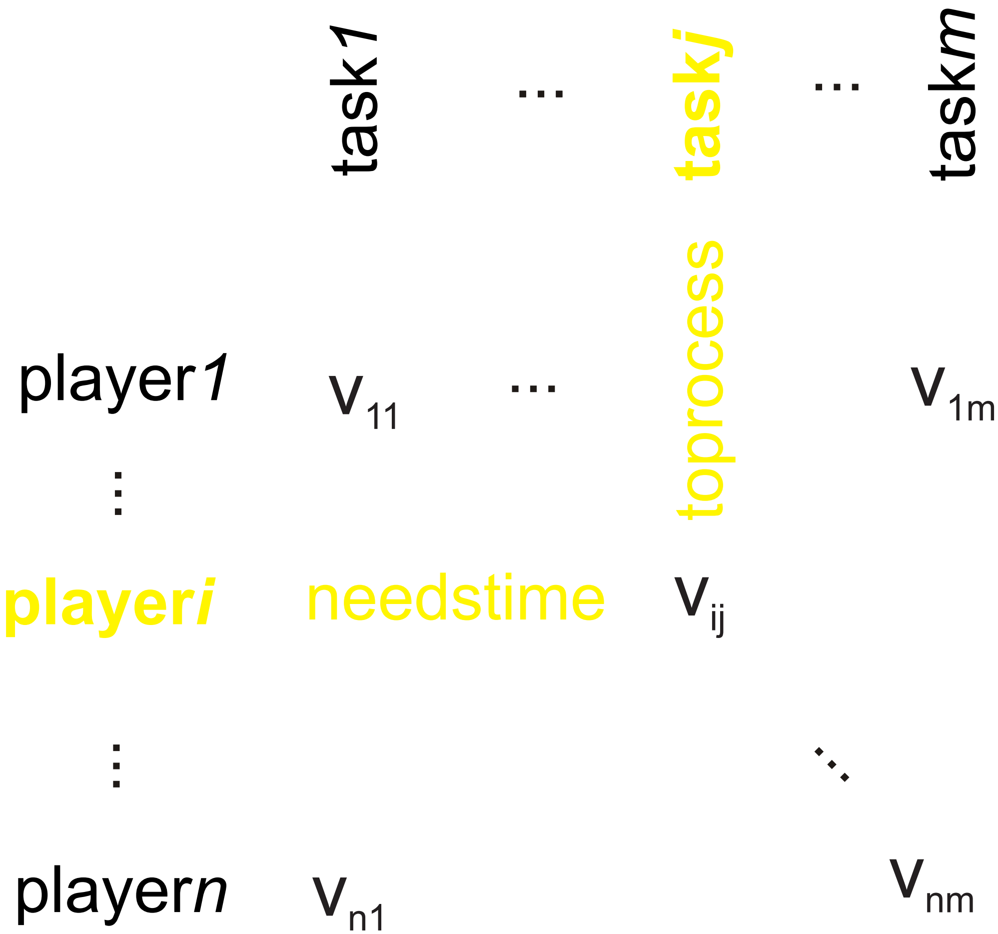
Output: allocation and payments

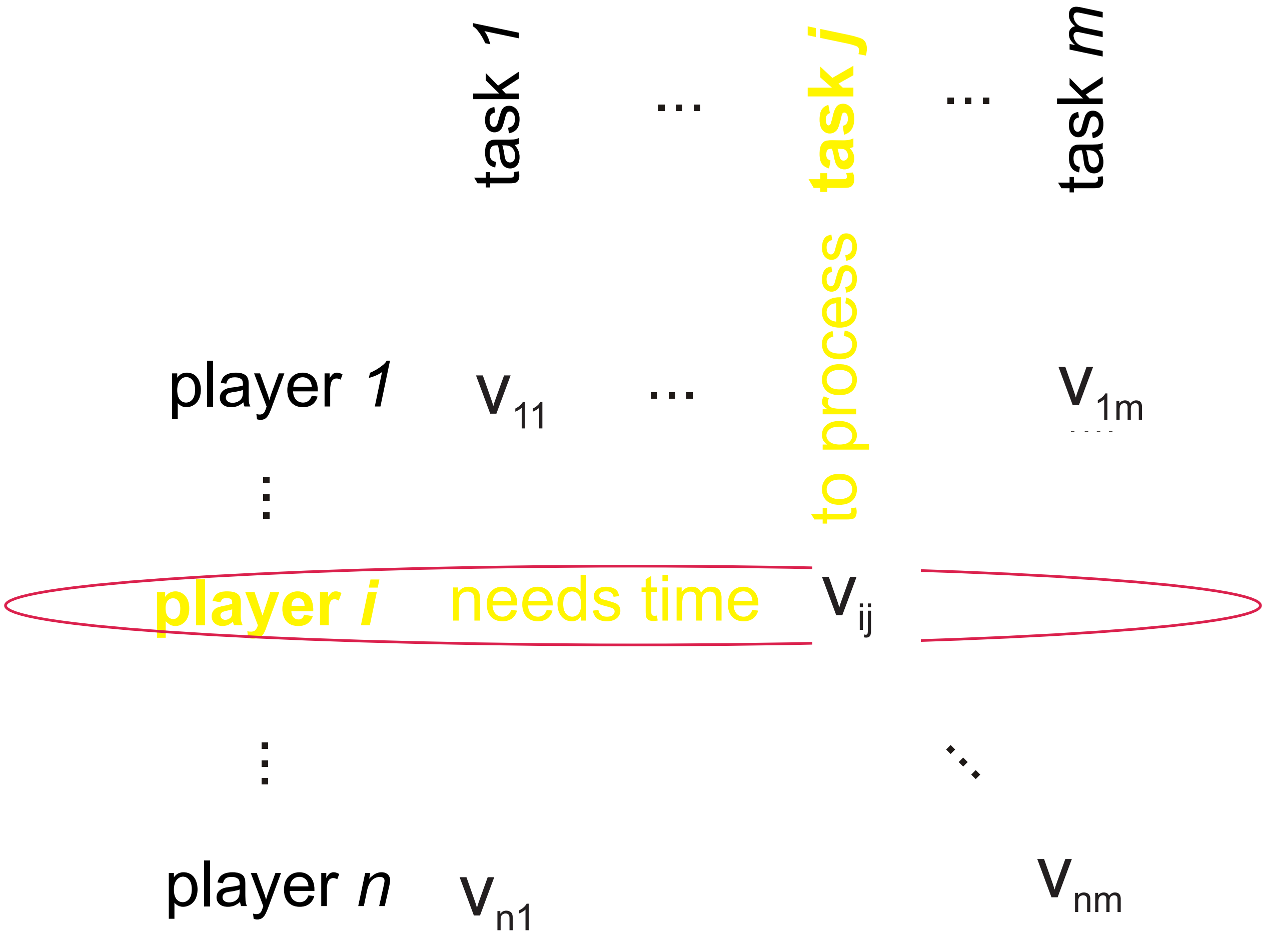
Scheduling unrelated Machines

It is a well-studied NP-hard problem. Lenstra, Shmoys, and Tardos showed that its poly-time approximation is between 3/2 and 2.

Nisan and Ronen in 1998 initiated the study of its mechanism-design version.

Archer and Tardos considered the mechanism-design version of the related machines problem. In this case, for each machine there is a single value (instead of a vector), its speed.


task1
...
taskj
...
taskm
toprocess
player1
v11
...
v1m
playeri
needstime
vij
playern
vn1
vnm



Only player i knows the values of his line. He can report a false value!

allocation $a_{ij} \in \{0,1\}$

Truthful

for fixed values of the other players

Player i doesn't have an incentive to lie.

v_i: valuation

v_{-I}: valuations of the other players except for player I (input)

a_i: allocation

p_i: payment of player I (output)

Selfish players want to maximize their utility: $p_i(a_i,v_{-i})-v_i\, a_i$

A mechanism is truthful if and only if for all v_i ,v_i'

:

$$p_i(a_i,v_{-i})-v_i\, a_i \geq p_i(a_i',v_{-i})-v_i\, a_i'$$

Weak-Monotonicity (W-mon)

is nesessary and sufficient for Truthfulness

the *row vectors of player i* satisfy

$$(\mathbf{a_i} - \mathbf{a_i'}) \cdot (\mathbf{v_i} - \mathbf{v_i'}) \leq 0.$$

- The other rows do not have to satisfy any condition

Strong-Monotonicity (S-mon)

the *row vectors of player i* satisfy

$$(\mathbf{a_i} - \mathbf{a_i'}) \cdot (\mathbf{v_i} - \mathbf{v_i'}) < 0.$$

for $v_i \neq v_i$' and $a_i \neq a_i$'.

- The other rows do not have to satisfy any condition

Parallels Arrow's IIA and non-bossiness

Independence of Irrelevant Alternatives (IIA)

If A is preferred to B out of the choice set {A,B},

introducing a third option X,
expanding the choice set to {A,B,X},

must not make B preferable to A.

used in Arrow's impossibility theorem 1950

S-Mon can be assumed w.l.o.g.:

- Unrestricted domain (Robert's Theorem):
- 2-pllayer case (except for tie-breaking)

Many of the known characterization results use it e.g. for combinatorial auctions Lavi Mu'halem Nissan [FOCS'03] Dobzinski Sundararajan [EC'08]

Is it restrictive?
Yes, very! But characterization proofs are complicated even after assuming it
IIA has an economical interpretation.

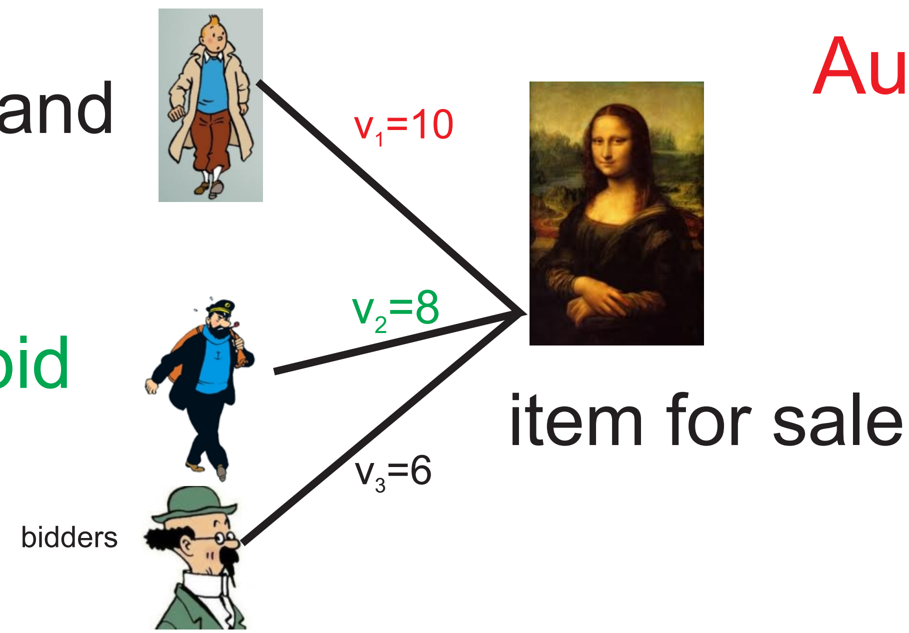
The Vickrey mechanism

maximizes welfare

Auction

The highest bid v_i wins and

pays 2nd highest bid



- ☑ truthful
- ☑ applies/can be generalized to many settings

- ☒ sometimes computationally inefficient
- ☒ doesn’t perform well if the objective isn’t welfare

Affine Maximizers

generalization of the Vickrey Mechanism

The Vickrey Mechanism selects the allocation $a \in [0,1]$ which maximizes the social welfare: $\Sigma_i a_i v_i$

An Affine maximizer selects the allocation a which maximizes the weighted social welfare $\Sigma_i \lambda_i a_i v_i + \gamma_a$
where $\lambda_i > 0$ (one for each player i) and
γ_a (one for each possible allocation) are constants.

Example of an affine minimizer:

$$\min\{v_{11}+v_{12}+1, v_{11}+v_{22}+2, v_{21}+v_{12}+5, v_{21}+v_{22}\}$$

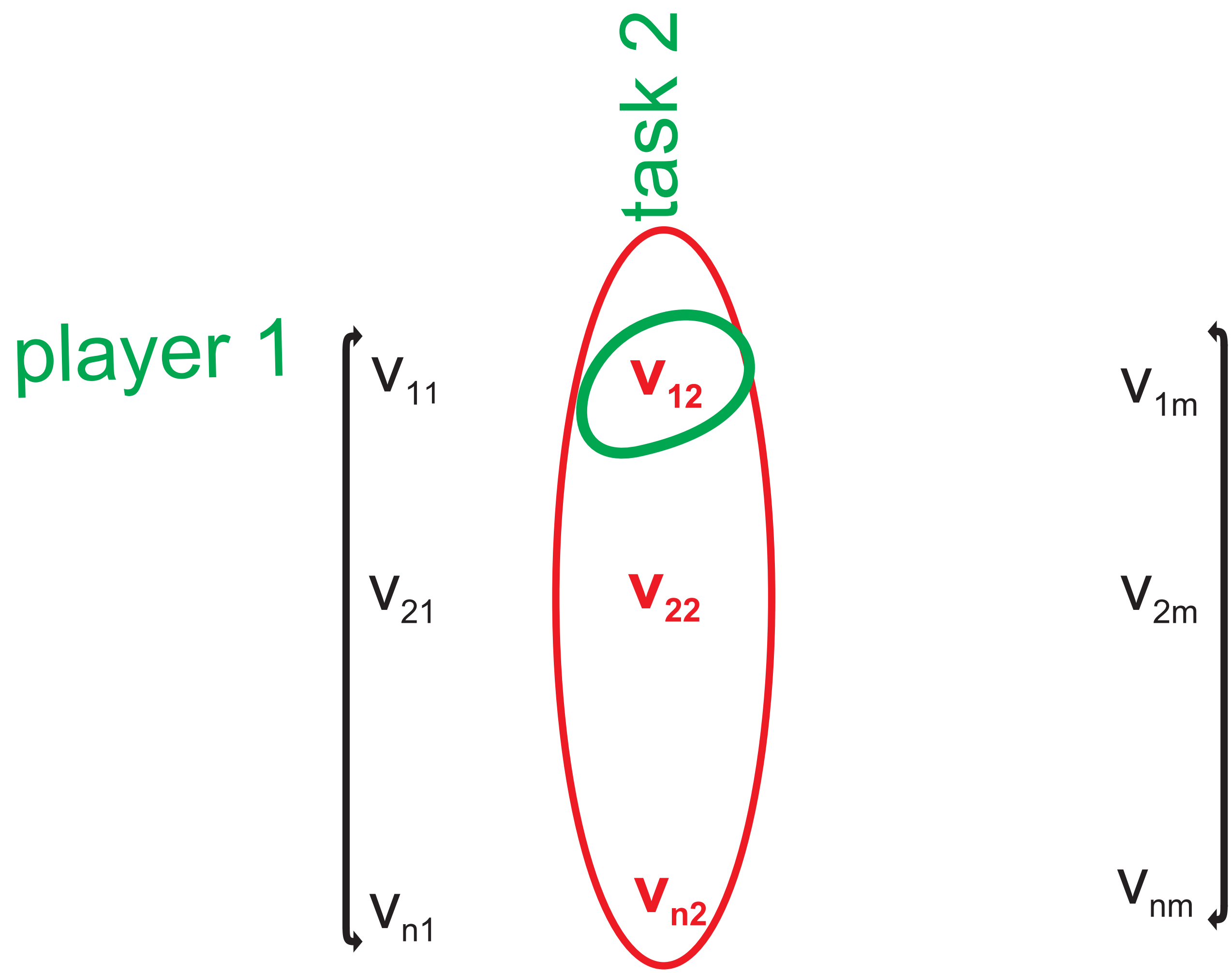
11	10	01	00
00	01	10	11

Input: $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

Task-independent mechanisms

only exist for the case of additive valuations

Which values determine the allocation a_{12}?



Allocate each item independently

Gibbard-Satterthwaite theorem for voting rules (1973)
For 3 or more outcomes, the only truthful mechanism is dictatorship.

Roberts theorem (1979)
For 3 or more outcomes, allowing payments, if we suppose that the domain of valuations is unrestricted the only truthful mechanisms are the affine maximizers.

2-player characterization

Theorem
The decisive truthful mechanisms for 2 players and 2 tasks are either affine maximizers or threshold mechanisms.

Theorem (2-player characterization)
[Christodoulou, Koutsoupias, Vidali ESA'09]
The decisive truthful mechanisms for m tasks partition the tasks into groups such that every group is allocated either by an affine maximizer or by a threshold mechanism.

Is the characterization the same for:

- ☐ n-player mechanisms
- ☐ n-player S-Mon mechanisms

?

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No: Grouping minimizers!

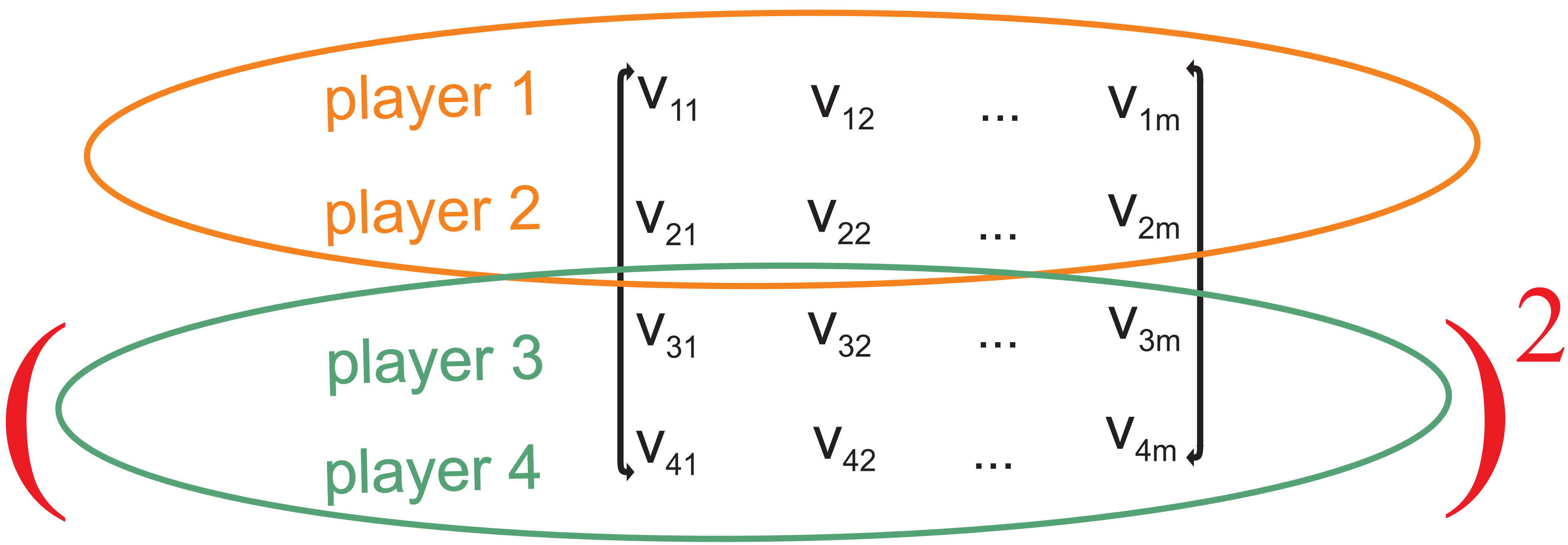
Grouping Minimizers

1. Run Affine minimizer (λ,γ) for players 1 and 2
2. Run Affine minimizer (λ',γ') for players 3 and 4
3. The different groups of players compete which group is getting the tasks:

Compute $\min\left\{\sum_{i=1,2}\lambda_i a_i v_i+\gamma_a,\ \left(\sum_{i=3,4}\lambda_i' a_i' v_i+\gamma_a'\right)^2\right\}$

where a,a' the winning allocations of each affine minimizer.

(instead of x^2 you can use any increasing bijection)



Theorem
The truthful scheduling mechanisms for n players and two tasks are either grouping minimizers or task-independent mechanisms.

Assumptions:
- decisiveness,
- S-Mon,
- boundaries are continuous functions (of the other players' bids)

The result extends to (subadditive, superadditive, submodular) combinatorial auctions that allocate all items! ([Vidali '11])