A characterization of n-player strongly monotone scheduling mechanisms

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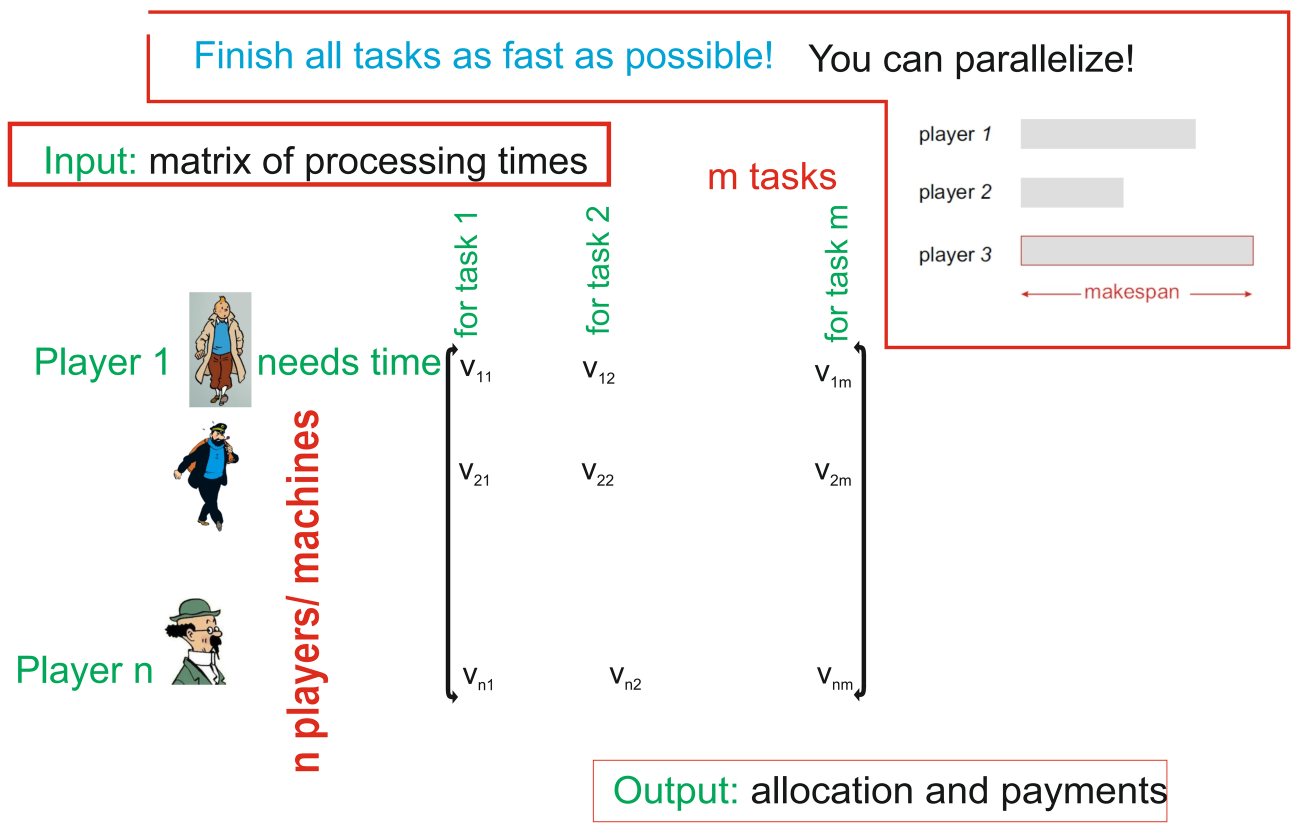
Scheduling Selfish unrelated Machines



It is a well-studied NP-hard problem. Lenstra, Shmoys, and Tardos showed that its poly-time approximation ratio is between 3/2 and 2.

Nisan and Ronen in 1998 initiated the study of its mechanism-design version.

Truthfulness: No player can increase his utility by lying.

Selfish players want to maximize their utility: $p_i(a_i, v_{-i}) - v_i\, a_i$

v_i: valuation

v_{-i}: valuations of the other players except for player I (input)

a_i: allocation

p_i: payment of player I (output)

A mechanism is truthful if and only if for all v_i, v_i':

$$p_i(a_i, v_{-i}) - v_i\, a_i \geq p_i(a_i', v_{-i}) - v_i\, a_i'$$

Weak-Monotonicity (W-mon) is nesessary and sufficient for Truthfulness

the *row vectors of player i* satisfy

$$(\mathbf{a_i} - \mathbf{a_i'}) \cdot (\mathbf{v_i} - \mathbf{v_i'}) \leq 0.$$

- The other rows do not have to satisfy any condition

Strong-Monotonicity (S-mon)

the *row vectors of player i* satisfy

$$(\mathbf{a_i} - \mathbf{a_i'}) \cdot (\mathbf{v_i} - \mathbf{v_i'}) < 0.$$

for $v_i \neq v_i'$ and $a_i \neq a_i'$.

- The other rows do not have to satisfy any condition

Strong-Monotonicity parallels Arrow' IIA

Independence of irrelevant Alternatives (IIA)

If A is preferred to B out of the choice set {A,B},

introducing a third option X,
expanding the choice set to {A,B,X},

must not make B preferable to A.

used in Arrow's impossibility theorem 1950

S-Mon can be assumed w.l.o.g.:
- Unrestricted domain (Robert's Theorem):
- 2-pllayer case (except for tie-breaking)

Many of the known characterization results use it e.g.
for combinatorial auctions Lavi Mu'halem Nissan [FOCS'03]
Dobzinski Sundurarajan [EC'08]

Is it restrictive?
Yes, very! But characterization proofs are complicated even after assuming it
IIA has an economical interpretation.

Affine Maximizers

The Vickrey Mechanism selects the allocation a which maximizes the social welfare: $\sum_i a_i v_i$

An Affine maximizer selects the allocation a which maximizes the weighted social welfare $\sum_i \lambda_i a_i v_i + \gamma_a$

where $\lambda_i > 0$ (one for each player i) and

γ_a (one for each possible allocation) are constants.

(there exist payments that make the mechanisms truthful)

Example of an affine minimizer:

$$\min\{v_{11}+v_{12}+1,\ v_{11}+v_{22}+2,\ v_{21}+v_{12}+5,\ v_{21}+v_{22}\}$$

11	10	01	00
00	01	10	11

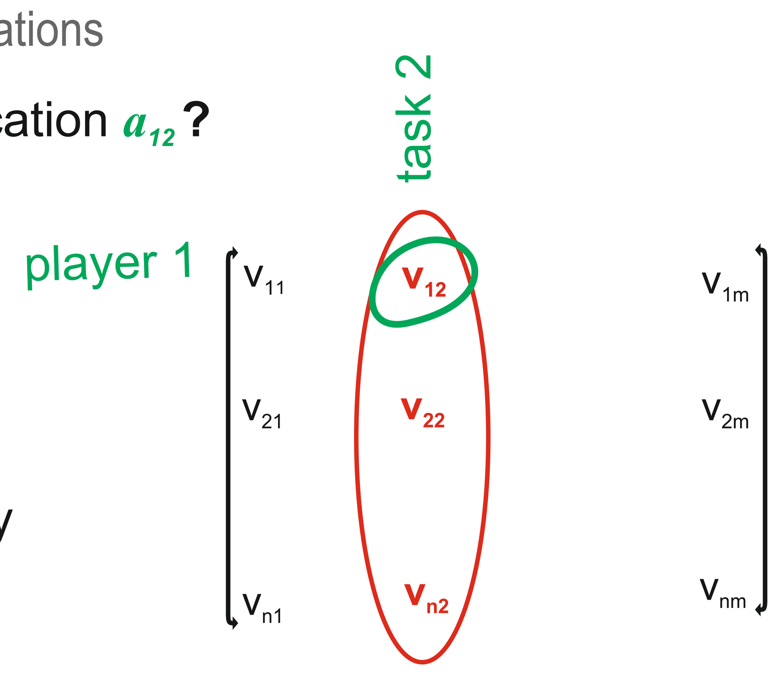
Input: $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

Task-independent mechanisms

only exist for the case of additive valuations

Which values determine the allocation a_{12}?

Allocate each item independently



Gibbard-Satterthwaite theorem for voting rules (1973)
For 3 or more outcomes, the only truthful mechanism is dictatorship.

Robert's theorem (1979)
For 3 or more outcomes, allowing payments, if we suppose that the domain of valuations is unrestricted the only truthful mechanisms are the affine maximizers.

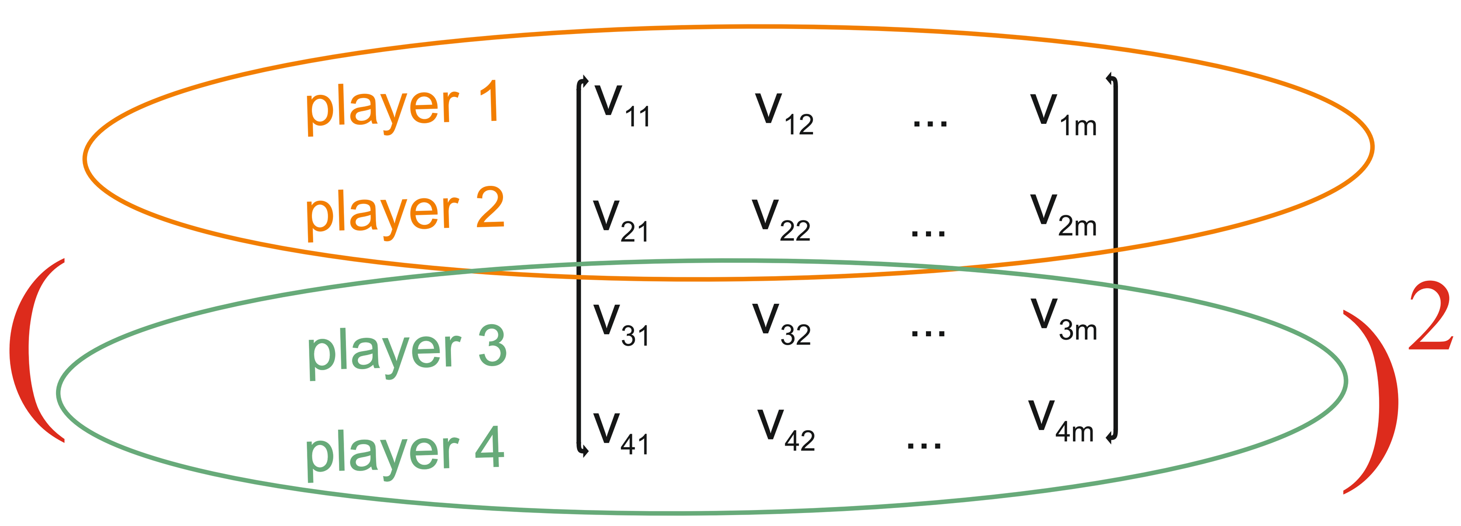
Grouping Minimizers

1. Run Affine minimizer (λ,γ) for players 1 and 2
2. Run Affine minimzer (λ',γ') for players 3 and 4
3. The different groups of players compete which group is getting the tasks:

Compute $\min\left\{\sum_{i=1,2} \lambda_i a_i v_i + \gamma_a,\ \left(\sum_{i=3,4} \lambda_i' a_i' v_i + \gamma_a'\right)^2\right\}$

where a, a' the winning allocations of each affine minimizer.

(instead of x^2 you can use any increasing bijection)



Theorem
The truthful scheduling mechanisms for n players are either grouping minimizers or task-independent mechanisms.

Assumptions:
- decisiveness,
- S-Mon,
- boundaries are continuous functions of other player's bids

The result extends to (subadditive, superadditive, submodular) combinatorial auctions that allocate all items! ([Vidali '11])