

Multi-parameter mechanism design under budget and matroid constraints

Monika Henzinger Angelina Vidali

University of Vienna, Department of Informatics

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Bayesian vs. Worst case

- I dont understand why you keep someone around your business and you cant even trust em...
- I ain't gotta trust her. I know her...
- I don't know what that means, man...
- Well, you can't trust Melanie...
- but you can always trust Melanie to be Melanie...

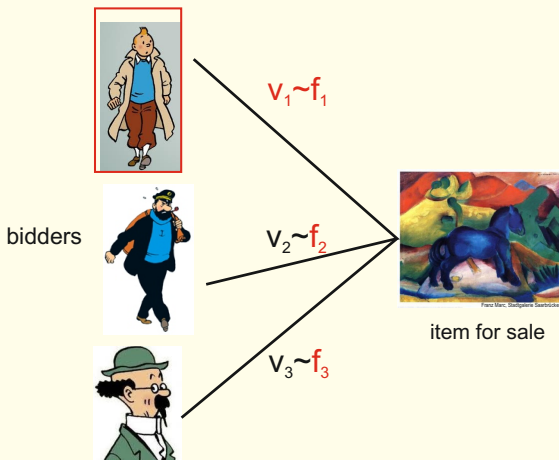


[Quentin Tarantino, Jackie Brown, 1997]

Moral: If you know the distributions of the players
it is easier to design a truthful mechanism.

Bayesian Truthful

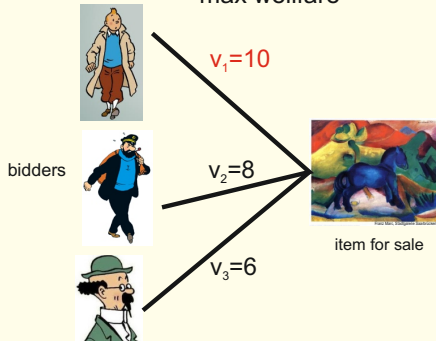
Truth-telling (reporting v_i) maximizes each bidder's expected utility.
(The expectation is taken over the distributions of the other bidders f_{-i} .)



The distributions of all bidders $f_1, \dots, f_n$ are public. The valuations private.

The Vickrey mechanism

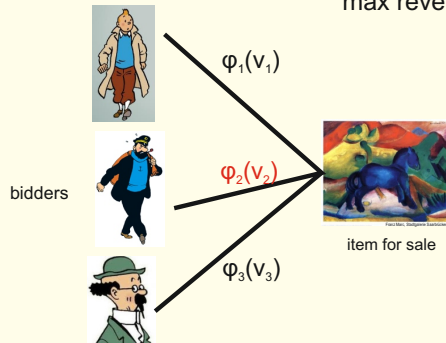
max welfare



The bidder with the highest valuation v_i wins

Myerson's optimal mechanism

max revenue



The bidder with the highest (positive) **virtual** valuation $\phi(v_i)$ wins

Hazard rate of the distribution: $h(v) = \frac{1-F(v)}{f(v)}$

Virtual valuation: $\phi(v) := v - h(v)$

Myerson '81, Optimal Bayesian mechanism design

- a single item for sale, n bidders
- player i has a private valuation v_i ,
- drawn from a publicly known distribution i.e. $v_i \sim f_i$
- Goal: **maximize the revenue** of the auctioneer

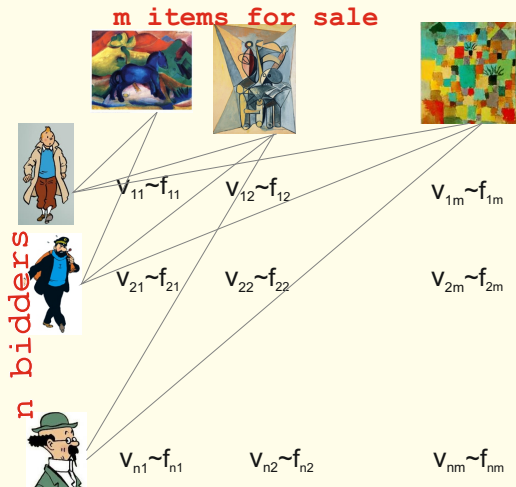
For all i ,

$$E[p_i] = E[x_i \cdot \phi_i(v_i)].$$

$$E[\text{payment}] = E[\text{allocation} \times \text{virtual valuation}]$$

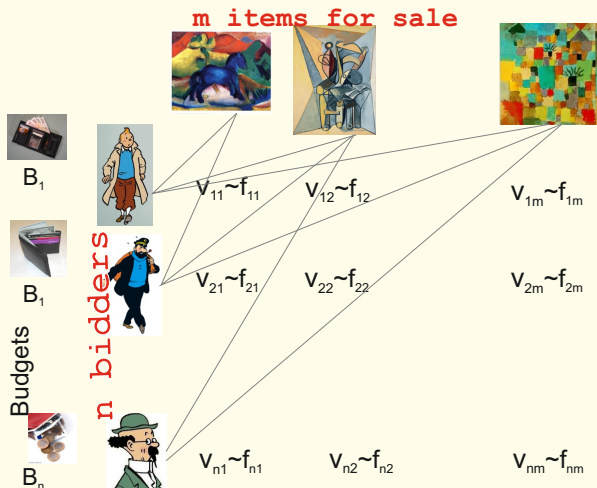
The optimal mechanism is the **Vickrey mechanism applied on the virtual valuations**.

Which auction maximizes the revenue for multi-parameter settings?



no externalities: the valuations of different bidders are independent.

Which auction maximizes the revenue for multi-parameter settings?



no externalities: the valuations of different bidders are independent.

Which is the optimal (/approximately optimal) mechanism?

Combinatorial Auctions

- n bidders, m items

Bayesian

- Each bidder i has a private valuation v_{ij} for item j , $v_{ij} \in [1, L]$, $v_{ij} \sim f_{ij}$. Additionally we assume that the input is rational numbers.
 - the distributions of the same player for different items are **correlated**
 - the distributions of the same player for different items are **independent**

Additional Budget and Feasibility Constraints

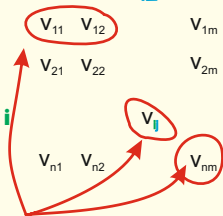
- a hard public budget constraint B_i for each player i .
- and a matroid constraint on the items a player can pick
- (a player can get multiple items)

Matroid Constraints we consider

Global matroids

item j

bidder i

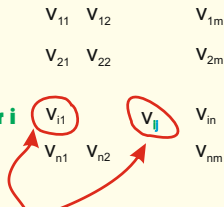


set of allocated items:
independent set of the matroid

Individual matroids

item j

bidder i



set of allocated items to player i :
independent set of the matroid M_i

Matroid Constraints we consider

- Let S_i be the set of items allocated to player i
- then $S := \cup_i S_i$ is the set of items allocated

Global matroid constraints

There exists *one* matroid $\mathcal{M}$ such that S has to be an independent set in $\mathcal{M}$.

Individual Matroid Constraints

For each player i , there exists *one* matroid $\mathcal{M}_i$ such that S_i has to be an independent set in $\mathcal{M}_i$.

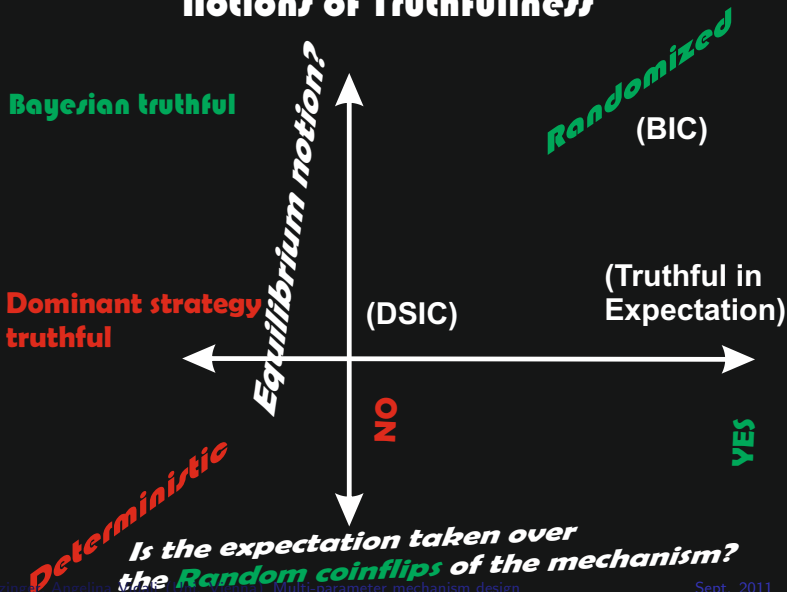
Different notions of truthfulness

Where do we allow randomization?

A mechanism is **truthful** when a player who reports his true valuations,

- maximizes his **utility**. (DSIC/Universally truthful)
- maximizes his **expected utility** over:
the valuation distributions of the players (Truthful in Expectation)
- maximizes his **expected utility** over:
the valuation distributions of the players **AND**
the random coinflips of the mechanism (BIC)

Randomized mechanisms: Notions of Truthfulness



Related work on multi-parameter settings [1/2]

- [Chawla, Hartline, Malec, Sivian STOC'10]
Approx. optimal mechanisms for various types of global matroids, polynomial in n, m , unit-demand, no budget constraints.
- [Bhattacharya, Goel, Gollapudi, Mungala STOC'10]
Approx. optimal mechanisms for Additive valuations, Budgets, only uniform Individual matroids, polynomial in n, m , and the support size of the valuation distributions.

Our results

Approx. optimal mechanisms various matroid constraints, budgets, additive valuations, polynomial in n, m , and the support size of the valuation distributions.

[Saeed Alaei, to appear in FOCS'11]

Improved approximation ratios for additive valuations and individual uniform matroids. A general technique to go from a 1-bidder mechanism to a mechanisms with n bidders by loosing only a factor of 2.

Related work on multi-parameter settings [2/2]

- [Dobzinski, Fu, Kleinberg STOC'11]
An **optimal** mechanism also for **correlated bidders without matroid or budget constraints**, **polynomial in n, m** , and the support size of the **valuation distributions**.
- [Papadimitriou, Pierrakos STOC'11]
An optimal mechanism for 2 correlated bidders, a hardness result for more. (single parameter)
- [Daskalakis, Weinberg '11] **PTAS** for **i.i.d. players** also for **correlated items**, **polynomial in n** for fixed m .

Correlated items [1]

Theorem (BGGM'10)

*For individual **uniform** matroid constraints, there exists a BIC, individually rational in expectation mechanism, whose revenue is a **4**-approximation to the optimal BIC mechanism.*

Theorem

*For individual **general** matroid constraints, there exists a BIC, individually rational in expectation mechanism, whose revenue is a **2**-approximation to the optimal BIC mechanism.*

Independently, different technique a 2-approximation but only for uniform matroids. [Saeed Alaei, to appear in FOCS'11]

Some key ideas

We write an LP similar to the one in [BGGM'10]

instead of writing the matroid constraint on expectation

$(\forall i, \forall S \subseteq J \quad \sum_{j \in S} \sum_r f_{ij}(r) x_{ij}(r) \leq \text{rank}(S))$ we do the following:

$$\forall i, t \quad \sum_{\text{independent } S} y_{iS}(t) \leq 1$$

$$\forall i, t, j \quad \sum_{\text{independent } S \text{ with } j \in S} y_{iS}(t) = x_{ij}(t)$$

$x_{ij}(t)$: probability that **item j is assigned to bidder i** when his type is t

$y_{iS}(t)$: probability that the **set S is assigned to bidder i** when his type is t .

Using the Ellipsoid method the LP can be solved in polynomial time:

The LP has an exponential number of variables, but the dual has only a polynomial number of variables and a polynomial time separation oracle.

The algorithm

The mechanism assigns the items to bidders as follows:

$A = J$

For $i = 1, 2, \dots, n$ % Bidders arrive sequentially.

Pick an indep. set T_i using the distribution $y_{S,i}(t_i)$;

% The distribution $y_{S,i}(t_i)$ tells us how to weight different independent sets.

set $S_i = \emptyset$

for each $j \in T_i$: if $j \in A$ then with probability $1/(2Z_{ij})$ do:

$S_i = S_i \cup \{j\}$; $A = A - \{j\}$

Bidder i gets S_i and pays $p_i(t_i)/2$.

We define $Z_{ij} = 1 - \sum_{i' < i} x_{i'j}(t_{i'})/2$, so that $P(j \in S_i) = x_{ij}(t_i)/2$.

% with constant probability we leave the item unallocated so if a bidder who comes later would pay more for it, it is still unallocated.

Independently distributed items the LP

In this section the distributions of player i for different items j are independent.

(LP2)[BGGM'10] Maximize $\sum_{i,j} \sum_r r f_{ij}(r) x_{ij}(r)$

$$\forall i, \forall S \subseteq J \quad \sum_{j \in S} \sum_r f_{ij}(r) x_{ij}(r) \leq \text{rank}(S) \quad (1)$$

$$\forall i \quad \sum_j \sum_r r f_{ij}(r) x_{ij}(r) \leq B_i \quad (2)$$

$$\forall j \quad \sum_i \sum_r f_{ij}(r) x_{ij}(r) \leq 1 \quad (3)$$

$$\forall i, j, r \quad x_{ij}(r) \in [0, 1] \quad (4)$$

The LP has an exponential number of variables, but the dual has only a polynomial number of variables and a polynomial time separation oracle.

Independently distributed items

Theorem

If, $v_{ij} \in [1, L]$ follow independent distributions f_{ij} . Then there is a $O(\log L)$ approximation of the revenue of the optimal BIC mechanism through a sequential posted price mechanism under any matroid constraint.

[BGGM'10] give a $O(\log L)$ approximation for k -uniform matroids.

How do we deal with general matroid constraints?

Idea: Offer each bidder i a set S of items, where each item has **the same ("average") price r_i** . Let the player pick an independent set, you don't lose more than $1/2$ the revenue you could get by choose a revenue maximizing independent set.

Can we do better with a sequential posted price mechanism?

No! [BGGM'10], not even if we assume regularity.

Independently distributed items

Theorem

Assume for all i and j , $v_{ij} \in [1, L]$ follow independent distributions f_{ij} . Then there is a $O(\log L)$ approximation of the revenue of the optimal BIC mechanism through a sequential posted price mechanism under any matroid constraint.

[BGGM'10] give a $O(\log L)$ approximation for k -uniform matroids.

How do we deal with general matroid constraints?

For k -uniform matroids you can argue about each (i, j) pair separately.

Idea: Offer each bidder i a set S of items, where each item has **the same** (“average”) **price** r_i . Let the player pick an independent set, you don't lose more than $1/2$ the revenue you could get by choosing a revenue maximizing independent set.

Can we do better with a sequential posted price mechanism?

No! [BGGM'10], not even if we assume regularity.

For MHR distributions LP3 $8e^2$ -approximation of opt

intuition: for single parameter settings welfare and revenue are within a factor e from each other

$$\begin{aligned}
 \text{(LP3)} \quad & \text{Maximize } \sum_{i,j} \sum_r f_{ij}(r) \phi_{ij}(r) x_{ij}(r) \\
 & \forall i, \forall S \subseteq J \quad \sum_{j \in S} \sum_r f_{ij}(r) x_{ij}(r) \leq \text{rank}(S) \\
 & \forall i \quad \sum_j \sum_r f_{ij}(r) \phi_{ij}(r) x_{ij}(r) \leq B_i \\
 & \forall j \quad \sum_i \sum_r f_{ij}(r) x_{ij}(r) \leq 1 \\
 & \forall i, j, r \quad x_{ij}(r) \in [0, 1]
 \end{aligned}$$

Two prices r_{ij}^* , $r_{ij}^* + 1$ are sufficient [BGGM'10]
to express the optimal revenue from LP3

The target revenue for the player-item pair (i, j) is

$$R_{ij} := p_{ij} r_{ij}^* P(\mathcal{V}_{ij} \geq r_{ij}^*) + (1 - p_{ij})(1 + r_{ij}^*) P(\mathcal{V}_{ij} \geq r_{ij}^* + 1)$$

Post price r_{ij}^* with probability p_{ij} and $r_{ij}^* + 1$ with probability $1 - p_{ij}$.
Let the player pick the independent set of items he wants.

Valuation distributions with Monotone Hazard Rate

Theorem

Suppose that $v_{ij} \in [1, L]$ follow *independent* f_{ij} satisfying the *monotone hazard rate condition*. Then for every bidder i there is an $O(\log(r_i^{\max}/r_i^{\min}))$ approximation of OPT_i under *any matroid constraint*.

Corollary

If $f_{ij}(r) \geq 1/m^c$ for all i, j , and r and some constant c , then our mechanism is an $O(\log m)$ approximation of the optimal revenue.

Special Matroids for which we get **constant approximation**

k-Uniform Matroid (/Capacity constraint):

A set is independent iff it contains $\leq k$ items.

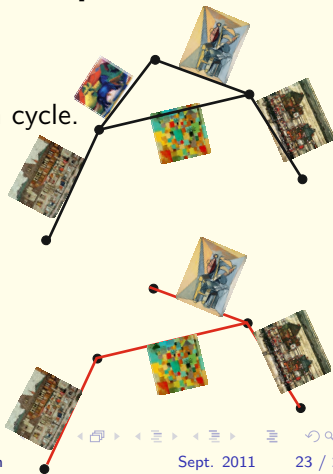
(/You cannot allocate more than k items.)

-We improve the approximation ratio from [BGGM'10].

Graphical Matroid:

There is a graph, with one edge for each item.

The set of allocated items should not contain a cycle.



A summary of our results for independent valuations

	Individual matroids	Global matroid
General matroids	$O(\log L)$	$O(\log L)$
Uniform matroids	(previous: $O(\log L)$ [BGGM10])	$O(\log L)$
General matroids & MHR	$O(\log m), O(k)$	$O(\log m), O(k)$
Uniform matroids & MHR	9, (previous: 24 [BGGM10])	9
Graphical matroids & MHR	16	64 with budgets and 3 without

All approximation ratios are with respect to LP3, LP3 is an $8e^2$ approximation to the optimal mechanism.